Title: Regular Polygons: Computing the Area

Link to Outcomes:

- **Reasoning** — Students will use inductive reasoning to derive the formula for finding the area of any regular polygon and will solve examples by using the formula.

- **Connections** — Students will be able to see a connection between area of a regular polygon, technological and real-world applications, and other disciplines.

- **Communications** — Students will work in groups to experiment with several regular polygons, looking for a pattern to form one formula for finding the area of any regular polygon. They will also explain their reasoning and the process that they used.

- **Technology** — Students will use a computer and *Geometer's Sketchpad*™ to find the area of a regular polygon.

- **Real-World Applications** — Students will use the formula to solve problems with real-world applications. Students will make up problems with real-world connections.

Brief Overview:

The formula for finding the area of a regular polygon is $A = 1/2\ aP$, where a is the apothem and P is the perimeter. This lesson is designed to challenge students to discover the formula for finding the area of a regular polygon and then to use the formula to solve problems involving the area of regular polygons.

Grade/Level:

Grades 9 – 12, Geometry

Duration/Length:

This lesson will take from 2 to 3 days depending on the level of students.

Prerequire Knowledge:

Students should:

- be familiar with the use of the *Geometer's Sketchpad*™.
- have knowledge of how to find the area of a triangle.
- be able to find the measure of each interior angle of a regular polygon.
- be able to determine how many triangles can be formed within a regular polygon by connecting the center to each vertex of the polygon.
- have knowledge of congruent triangles in order to establish a relationship among the triangles within the regular polygon.

Objectives:

Students will be able to:

- State general information about a regular polygon:
    - Define a regular polygon
    - Find the measure of each interior angle
    - Find the area of a triangle
- Derive the formula for the area of a regular polygon
- Solve problems involving area of a regular polygon

Materials/Resources/Printed Materials:

- Student Worksheets
- Compass or a protractor
- Ruler
- Overhead Projector
- Transparencies
- Transparency pens
- Calculator
- Computer
- *Geometer's Sketchpad*™ Software

Development/Procedures:

1. Divide the class into groups of four or five and lead the class through a review of regular polygons.

    - Define a regular polygon.
    - Show pictures of regular polygons (visual A).
    - Discuss how to determine the measure of each interior angle of a regular polygon (visual B).
    - Distribute to each group worksheet 1.
    - Instruct students to work in groups to complete worksheets. Tell students that they have 8 minutes to complete this worksheets.
    - Let groups present, compare, and discuss results.

2. Distribute to each group a copy of visual C. Work through the first example to explain the procedure.

    (Place instructions on the board.)
    - Bisect each angle of the polygon.
    - Locate the point of intersection of each bisecting ray and call it your center.
    - Name the triangles formed ΔA, ΔB, ΔC, … depending on how many triangles are formed.
    - Show that $\Delta A \approx \Delta B \approx \Delta C$ ….
    - Find the area of ΔA.
    - Find the area of $\Delta A + \Delta B + \Delta C$ ….
    - Find the area of the polygon.

3. Instruct each group to complete the worksheets following the seven steps illustrated. After about 10 minutes, allow the group reporters to present their results to the class.

4. Instruct the students to work in groups to try to come up with a general formula for finding the area of a regular polygon.

5. Have group reporters present their conjectures of a general rule for finding the area of a regular polygon. Questions such as the following might be asked.

   - What do you think is the rule for finding the area of a regular polygon?
   - Why do you think that?
   - Does someone (group) have a different conjecture?
   - How do the conjectures that we have compare?
   - Can we agree on a formula for finding the area of a regular polygon? *(The results of this discussion should be* $A = 1/2\ aP$, *where* a = *apothem, the distance from the center to a side of the polygon and* P = *perimeter, the distance around a polygon.)*

6. Ask the class, "Now that we have derived a formula for finding the area of a regular polygon, what are we going to do with it?", or ask "Where might we use this?" (You might allow students to brainstorm in their groups.)

7. Take class responses, then show the problems for real world applications. (Show visual D, three examples of real world applications.)

8. If time permits, solve one of the examples.

9. Allow 3 to 4 minutes for closure.

10. Closure — Ask students, what did we do in class today?
    1. *Students discussed the regular polygon and finding the measure of each interior angle.*
    2. *Students derived the formula for finding the area of regular polygons.*
    3. *Students discussed real life applications of finding the area of regular polygons*

Evaluation:

Monitor each group as they work. Have each group make an oral presentation. Permit students/groups to ask questions.

Extension/Follow Up:

Distribute Worksheet 2 with some examples and application problems to solve as homework.

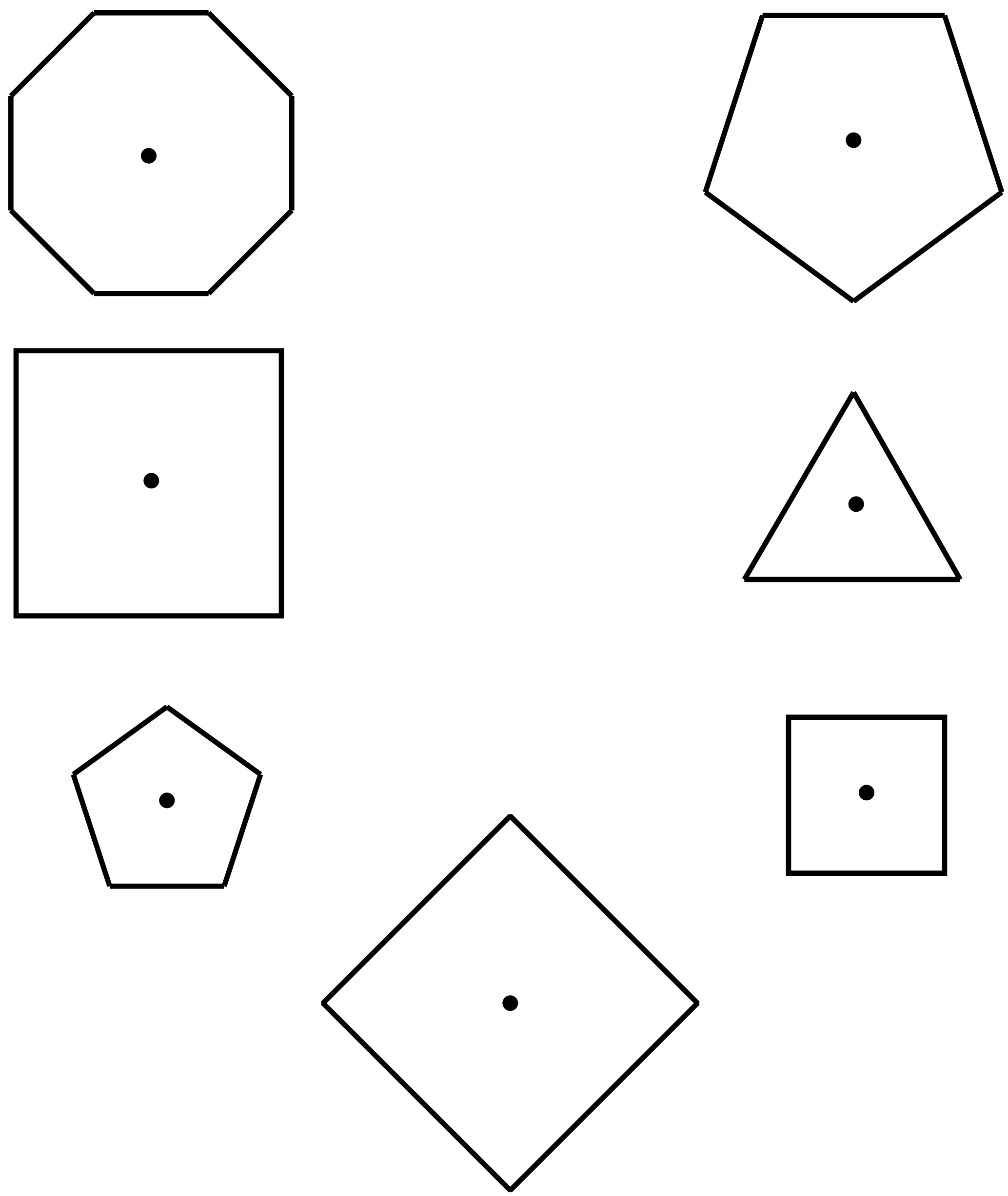
- The following day let students construct regular polygons using the *Geometer's Sketchpad*™. Compute the area, the apothem and the perimeter. Put the values of the apothem and perimeter into the formula, solve and compare the results with the area computed by the *Sketchpad*.

Authors:

Wanda G. Thomas
Broadneck Senior High School
Anne Arundel County

Hui Chong Seymour
Einstein High School
Montgomery County

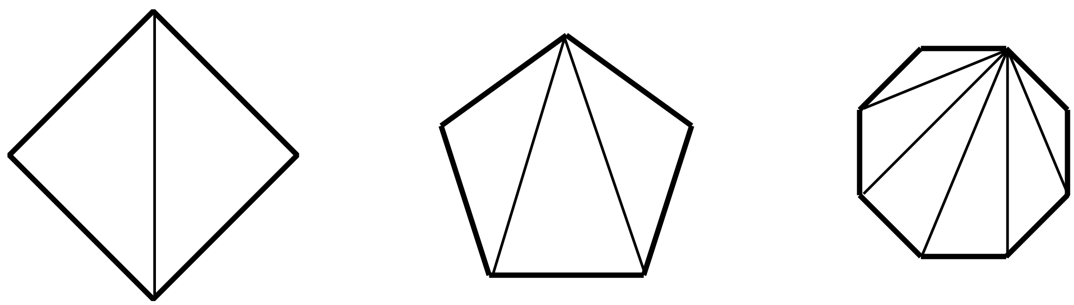
Visual A: Regular Polygons



Visual B: Angles of Polygons

The sum of the three interior angles of a triangle is 180°. This fact is called the "Angle Sum Theorem." Can this theorem be extended to other convex polygons? What is the interior angle sum of a stop sign (a regular octagon)?

The **diagonal** of a convex polygon is a line segment connecting any two non-adjacent vertices. In the three examples below, all possible diagonals have been drawn from one vertex.



The table below shows how to find the interior angle sum for any regular polygon by adding the angles of the triangles formed by all diagonals from one vertex.

Polygon	Number of Sides	Number of Triangles	Sum of Degree Measures of Angles	Measure of an Interior Angle
triangle	3	1	(1 • 180) or 180	180/3 or 60
quadrilateral	4	2	(2 • 180) or 360	360/4 or 90
pentagon	5	3	(3 • 180) or 540	540/5 or 108
hexagon	6	4	(4 • 180) or 720	720/6 or 120
heptagon	7	5	(5 • 180) or 900	900/7 or 128o
octagon	8	6	(6 • 180) or 1080	1080/8 or 135
nonagon	9	7	(7 • 180) or 1260	1260/9 or 140
decagon	10	8	(8 • 180) or 1440	1440/10 or 144
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
n-gon	*n*	*n* - 2	(*n* - 2)180	(*n* - 2)180/*n*

Since the angles in a regular polygon are congruent, you can find the measure of one interior angle of a regular polygon by dividing (*n* - 2)180 by the number of angles, n. The formula for this is, $\frac{(n - 2)180}{n}$.

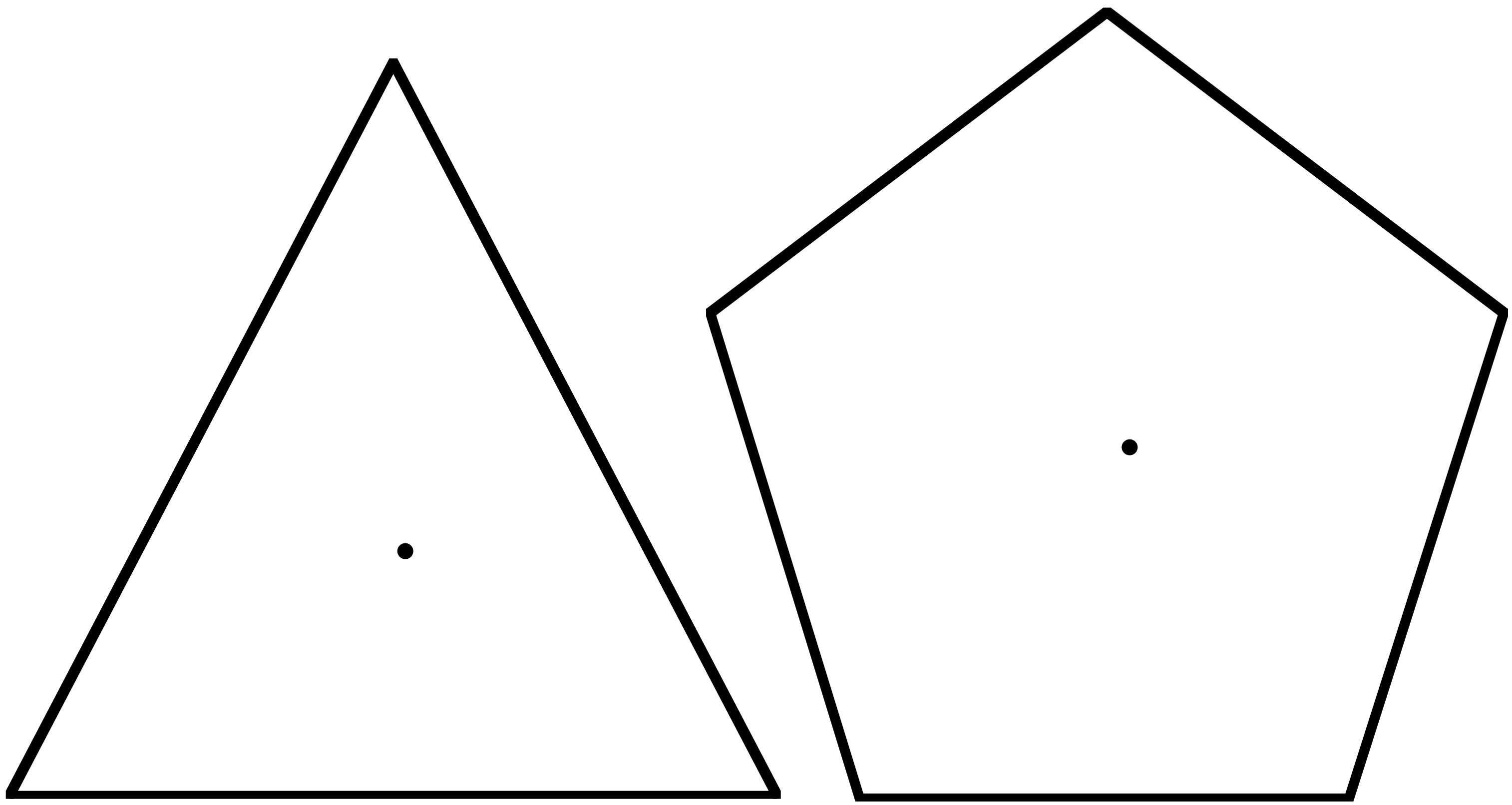
*Regular Polygons only.

Angles of Regular Polygons

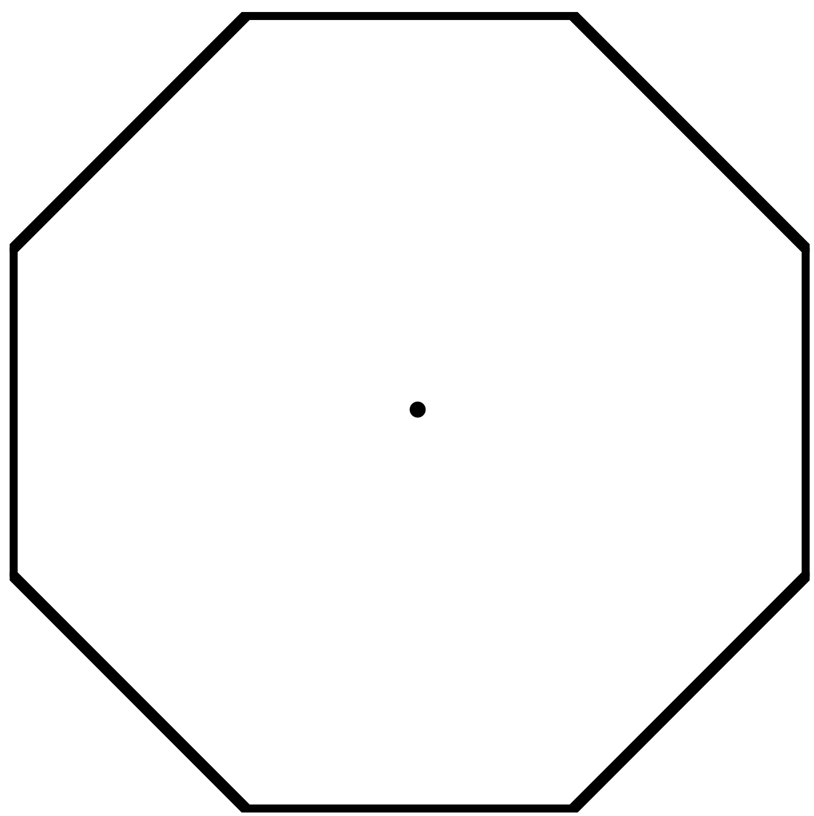
Complete.

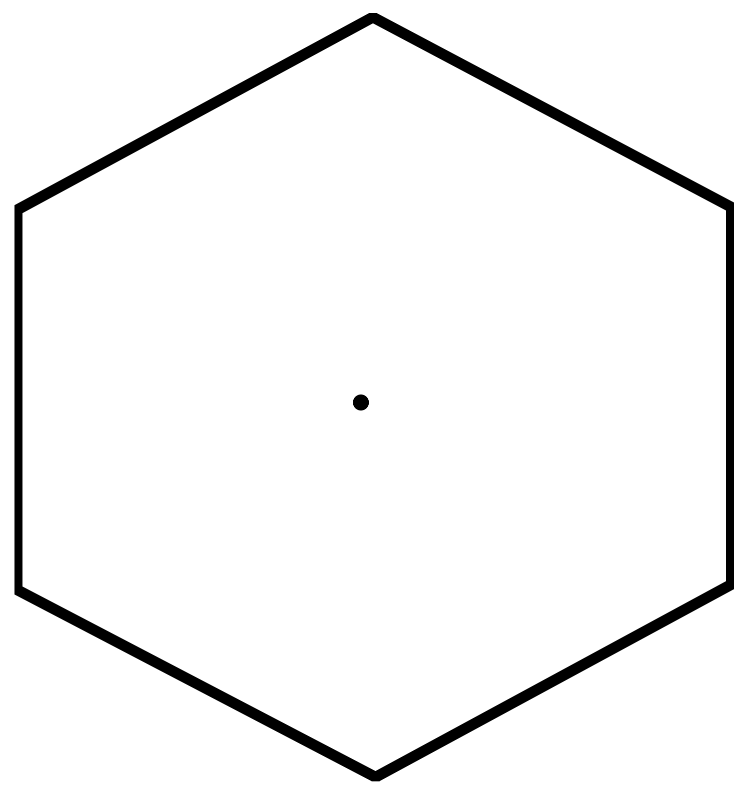
	Sum of Interior Angle Measures	Each Interior Angle Measure
1. Regular Octagon		
2. Regular Nonagon		
3. Regular 22-gon		
4. Regular 24-gon		
5. Regular Pentagon		
6. Regular Hexagon		
7. Regular 18-gon		
8. Square		

Visual C: Regular Polygons (Part I)



Visual C: Regular Polygons (Part II)





Visual D: Real-World Application

One of the most famous buildings in the United States is the Pentagon in Arlington, Virginia, where the Defense Department is located. This tremendous office complex has 17.5 miles of corridors. Each side of the building is 921 feet long, and the apothem of the pentagon is 633.8 feet.

1. What is the area (in square feet) of the Pentagon? What is the area in square yards?

2. How many square yards are there in a football field?

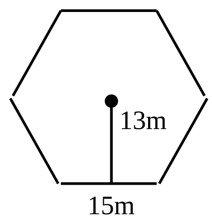
3. How many foot ball fields can you fit into the area of the Pentagon?

Name ___________________________ Worksheet 2

Area of a Regular Polygon

The **perimeter** of a polygon is the sum of the lengths of the sides. The **apothem** of a regular polygon is the distance from one side to the center. The **area of a regular polygon** is one-half of the product of the apothem and the perimeter.

Example:



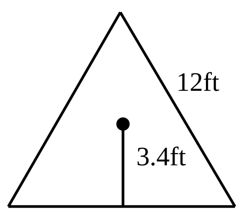
A = 1/2 ap

A = 1/2(13)(90)

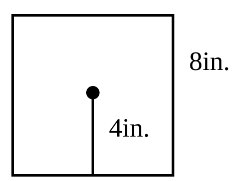
A = 585 m^2

Find the area of each regular polygon.

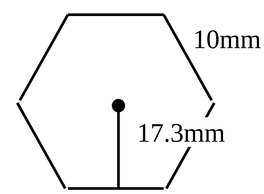
1.



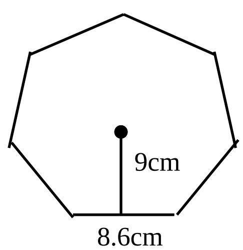
2.



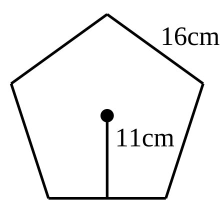
3.



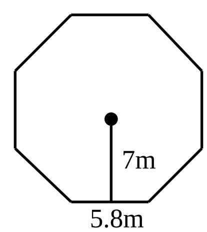
4.



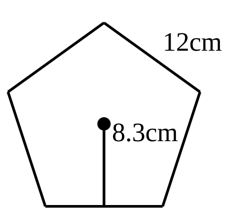
5.



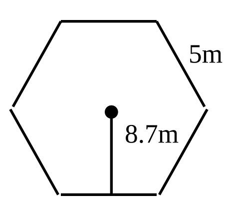
6.



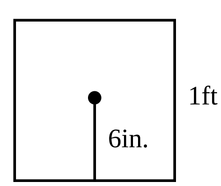
7.



8.



9.



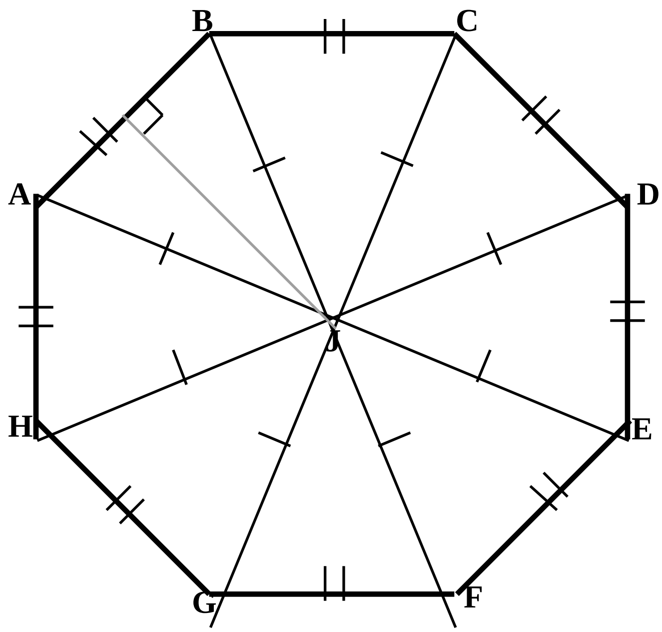
Solve.

10. Mrs. Smith wants to add a triangular deck in the yard behind her house. Each side is to be 18 feet long. Find the length of the railing that will fit completely around the deck. Then find the area of the deck.

11. Sam is building a pen for dogs near the barn. The pen will be a hexagon with sides of 3. Find the area of the pen.

12. What is the approximate footage enclosed at ground level of the Pentagon building in Washington, D.C. if its sides are about 280m and its apothem is about 193m?

Notes on Polygons for Visual C



1) $\Delta ABJ \cong \Delta BCJ \cong \Delta CDJ \cong \Delta DEJ \cong \Delta EFJ \cong \Delta FGJ \cong \Delta GHJ \cong \Delta HAJ$

2) Area of $\Delta ABJ = \frac{1}{2}bh$

3) Area $= \Delta ABJ + \Delta BCJ + \Delta CDJ + \Delta DEJ + \Delta EFJ + \Delta FGJ + \Delta GHJ + \Delta HAJ$

$$= \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh$$

$$= \frac{1}{2}(8bh) = \frac{1}{2}Pa$$

Note: A similar construction can be used for the triangle, pentagon, or hexagon on Visual C, or for any other regular polygon.

Answer Key
Angles of Regular Polygons

Complete.

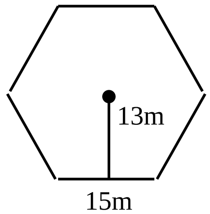
	Sum of Interior Angle Measures	Each Interior Angle Measure
1. Regular Octagon	(8-2)180 = 1080	1080/8 = 135
2. Regular Nonagon	(9-2)180 = 1260	1260/9 = 140
3. Regular 22-gon	(22-2)180 = 3600	3600/22 = 163.63
4. Regular 24-gon	(24-2)180 = 3960	3960/24 = 165
5. Regular Pentagon	(5-2)180 = 540	540/5 = 108
6. Regular Hexagon	(6-2)180 = 720	720/6 = 120
7. Regular 18-gon	(18-2)180 = 2880	2880/18 = 160
8. Square	(4-2)180 = 360	360/4 = 90

Name _____**Answer Key**________ Worksheet 2

Area of a Regular Polygon

The **perimeter** of a polygon is the sum of the lengths of the sides. The **apothem** of a regular polygon is the distance from one side to the center. The **area of a regular polygon** is one-half of the product of the apothem and the perimeter.

Example:



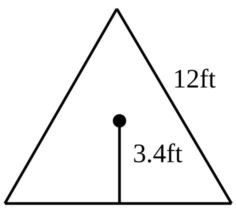
A = 1/2 aP

A = 1/2(13)(90)

A = 585 m^2

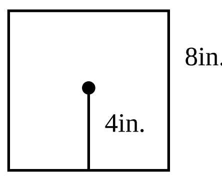
Find the area of each regular polygon.

1.



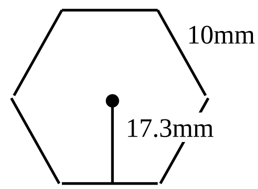
1/2 (3.4)(36) = 61.2 ft^2

2.



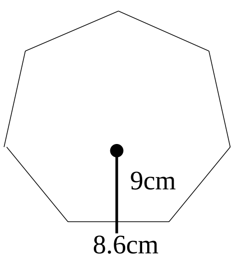
1/2 (4)(32) = 64 in^2

3.



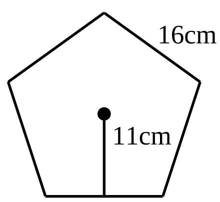
1/2 (17.3)(60) = 519 mm^2

4.



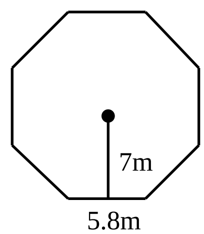
1/2 (9)(60.2) = 270.9 cm^2

5.



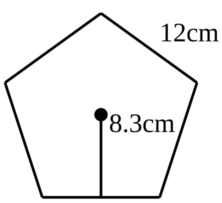
1/2 (11)(16) = 88 cm^2

6.



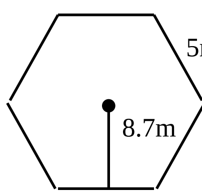
1/2 (7)(46.4) = 162.4 m^2

7.



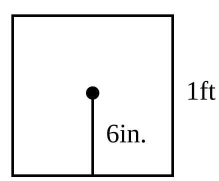
1/2 (8.3)(60) = 249 cm^2

8.



1/2 (8.7)(30) = 130.5 cm^2

9.



1/2 (6)(48) = 144 in^2

1/2 (1/2)(4) = 1 ft^2

Answer Key

Solve.

10. Mrs. Smith wants to add a triangular deck in the yard behind her house. Each side is to be 18 feet long. Find the length of the railing that will fit completely around the deck. Then find the area of the deck.

a) $3 \times 18 = 54$
b) $\frac{1}{2}(54)3\sqrt{3} = 81\sqrt{3}$ or ≈ 140.3

11. Sam is building a pen for dogs near the barn. The pen will be a hexagon with sides of 3. Find the area of the pen.

$$\textbf{Area} = \frac{1}{2}(18)\frac{3\sqrt{3}}{2} = \frac{27\sqrt{3}}{2} \text{ or } \approx 23.4$$

12. What is the approximate footage enclosed at ground level of the Pentagon building in Washington, D.C. if its sides are about 280m and its apothem is about 193m?

P = 5 x 280 = 1400
Area = 1/2 x (1400) x 193 = 135,100